Flux-freezing breakdown observed in high-conductivity magnetohydrodynamic turbulence

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Magnetic field lines in a resistive plasma "move" stochastically

\[ \partial_t \mathbf{u} = - (\mathbf{u} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \mathbf{u} - \nabla p + j \times \mathbf{b} + \mathbf{F} \]

\[ \partial_t \mathbf{b} = \nabla \times (\nu \nabla \mathbf{b}) \]

\[ j = \frac{\nu}{a} \nabla \times \mathbf{b} \]

Is standard flux-freezing valid for infinite conductivity?

Textbook derivations based on Alfvén's theorem fail when velocity gradients grow with increasing conductivity.

\[ \mathbf{b}(\mathbf{x}, t) = \frac{\mathbf{b}_0(\mathbf{a}) \cdot \nabla_s \mathbf{X}(\mathbf{a}, t)}{\det(\nabla_s \mathbf{X}(\mathbf{a}, t))} \mathbf{X}(\mathbf{a}, t) = \mathbf{x} \]

The usual derivation of the Lundquist formula assumes a smooth Lagrangian flow \( \mathbf{X}(\mathbf{a}, t) \) exists. In fact, unique trajectories for initial particle locations require finite velocity gradients:

\[ ||\mathbf{X}(\mathbf{a}, t) - \mathbf{X}(\mathbf{a}', t)|| \leq \exp(||\nabla \mathbf{u}||_{\infty}(t - t_0)||\mathbf{a} - \mathbf{a}'||) \]

MHD simulation data is stored in a public, web-accessible database, please visit http://turbulence.pha.jhu.edu for more information.

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