Exploring Finite Time Lyapunov Exponents in Isotropic Turbulence With the Johns Hopkins Turbulence Databases

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Johns Hopkins Turbulence Databases (JHTDB)

- http://turbulence.pha.jhu.edu/
- access via web services
- Fortran, C, Matlab, HDF5 cutout
- built-in functions
  - e.g. getVelocity, getPressureHessian
  - interpolation (time & space)
  - finite-differencing
- Currently hosts four datasets:
  - Isotopic: 1024^3
  - Magnetohydrodynamics: 1024^3
  - Channel: 2048 x 512 x 1536 x 1997
  - Mixing: 1024^3 x 1012

Homogeneous Isotropic Turbulence (HIT)

- forced incompressible Navier-Stokes equations in a (2π)^3 periodic box
- canonical problem for studying fluid turbulence

Finite-Time Lyapunov Exponents (FTLE)

- using velocity gradients, A_j = \partial u_j/\partial x_i, along fluid particle trajectories
- \text{dx/\partial t} = u(x,t), \quad x(t) = X, \quad \partial D/\partial t = AD, \quad \gamma(T; X, t_0) = \frac{1}{T} \ln(\sigma(T))
- exponential rate of separation of neighboring fluid particle trajectories
- deformation of small particles by turbulent flows
- identify coherent motions in fluid turbulence
- converge to the Lyapunov exponent, \gamma(T \rightarrow \infty) \rightarrow \lambda_f

Extracting FTLEs from the JHTDB HIT Simulation

- Initialize particle locations x(t_0) = X
- Loop through time:
  - Use getVelocityGradient database function to retrieve A_i(x, t) = \partial u_i/\partial x_k
  - Use getVelocity function to advance trajectory to next time step, x = u(x, t)
  - Use second simulation to advance D along each trajectory, D = AD
  - Periodically use Gram-Schmidt (QR decomposition) to compute orthogonal stretching rates
  - \gamma_i = \frac{1}{T} \ln(R_{ij})

Lagrangian Coherent Structures (LCS)

- technique for identification of coherent fluid motions
- attracting/repelling material surfaces
- ridges in the FTLE field \gamma(X, t_0) for a fixed integration time T
- JHTDB database example:

Large Deviation Formalism

- describes the behavior of the PDFs for sums of i.i.d. variables
- applies to FTLEs
- extends the central-limit theorem for \gamma(T \rightarrow \infty) by introducing the Cramér function, S(\gamma)

\[
p(\gamma, T) \sim \exp(-TS(\gamma))
\]

Effect of Rotation

- Advance deformation tensor with strain-rate tensor, D = SD

Large Deviations for Joint-Statistics

- the large deviation formalism is easily extended to joint statistics

\[
p(\gamma_i, T) \sim \exp(-TS(\gamma_i, \gamma_j))
\]

Conclusions

- {\lambda_1, \lambda_2, \lambda_3} = {0.114, 0.029, -0.143}, \lambda_1 : \lambda_2 : \lambda_3 \approx 4 : 1 : -5
- Bias for both weakly and strongly deformed particles: \gamma_2 > 0.
- Without particle rotation, deformation approximately doubled.